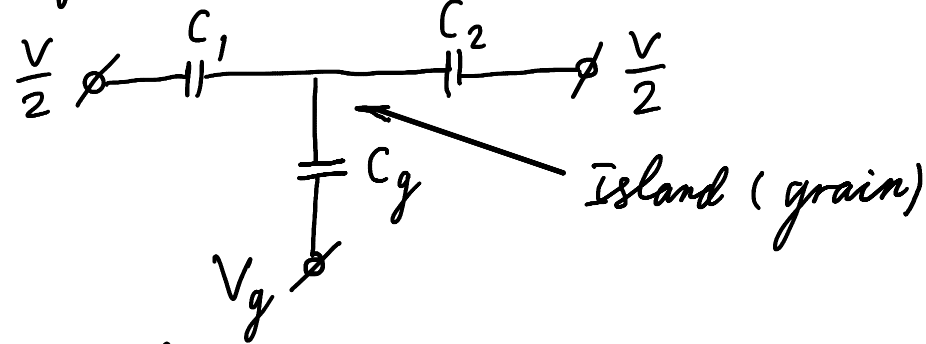
Parity effect in superconductors

At low temperatures, $T \ll T_c$, almost all electrons in a superconductor are paired and form Cooper pairs. What if the superconductor is finite (= superconductive grain)? The number of electrons may be even or odd. In the case of an odd number, there will remain one electron unpaired.

→ There may be a difference between systems with odd and even numbers of electrons

<u>Reminder</u>: quasiparticle energies $E_{k\alpha} = \sqrt{\Delta^2 + \varepsilon_{k\alpha}^2}$ ($\varepsilon_{k\alpha}$ measured from the chemical potential) are separated by the gap Δ from the condensate.

single-electron transistor



The charging energy

$$U = \frac{Q^2}{2C} = \min_n \frac{(en + C_g V_g)^2}{2C}$$

where $C = C_1 + C_2 + C_g$

→ in a normal system the conductance will be periodic as a function of $C_g V_g$ with the period e

However ... if the island is superconductive, then the periodicity is $2e$, because for states with an odd number of electrons there is an extra energy cost of Δ $(\gg E_C)$

A superconductive grain = a grand-canonical ensemble with a fixed parity

$$\text{Odd)}\quad Z_{odd} = \sum_i e^{-\beta E_i} + \frac{1}{6} \sum_i \sum_{j \neq i} \sum_{k \neq i,j} e^{-\beta (E_i + E_j + E_k)} + \ldots$$

$$\text{Even)}\quad Z_{even} = 1 + \frac{1}{2} \sum_i \sum_{j \neq i} e^{-\beta (E_i + E_j)} + \ldots$$

$$\Omega_{odd/even} = -T \ln Z_{odd/even} =$$

$$= -T \ln \left[\frac{1}{2} \prod_i (1 + e^{-\beta E_i}) \mp \frac{1}{2} \prod_i (1 - e^{-\beta E_i}) \right] =$$

$$= \underbrace{-T \ln \prod_i (1 + e^{-\beta E_i})}_{= \Omega_{BCS}} \underbrace{- T \ln \frac{1}{2} \left(1 \mp \prod_i \tanh \frac{\beta E_i}{2}\right)}_{\text{The contribution of the parity}}$$

The difference between odd and even numbers of electrons

$$\delta \Omega = \Omega_{odd} - \Omega_{even} = -T \ln \frac{1 - f(T)}{1 + f(T)}$$

for $f(T) = \prod \tanh \frac{\beta E_k}{2}$

$\delta\Omega = \Omega_{odd} - \Omega_{even}$

where $f(T) = \prod_{k\alpha} \tanh \frac{\beta E_k}{2}$

Compute $f(T)$ for a normal metal:

$$f(T) = \prod_{k\alpha} \tanh \frac{\beta|\xi_k|}{2} = e^{\sum_{k\alpha} \ln \tanh \frac{\beta|\xi_{k\alpha}|}{2}} =$$

$$= \exp\left(4\nu_0 V \int_0^\infty d\xi \ln \tanh \frac{\xi}{2T}\right) = e^{-2\pi^2 \nu_0 V T}$$

Degeneracy due to the spin and electrons / holes

For a normal metal, $\Omega_{odd} - \Omega_{even} \approx 2T e^{-2\pi^2 \nu_0 V T}$

Parity effects will manifest themselves at temperatures $T < T^*$, where

$$T^* \approx \frac{1}{2\pi^2 \nu_0 V} \lesssim 10^{-4}\,K$$

For a <u>superconductor</u>

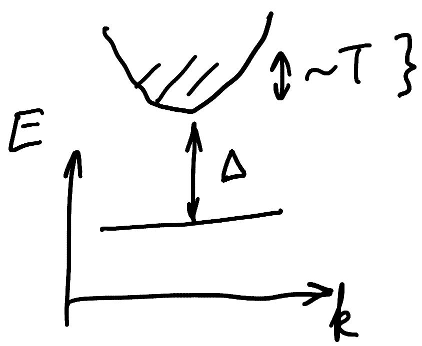
$$f(T) = \prod_{k\alpha} \tanh \frac{\beta E_{k\alpha}}{2} =$$

$$= \exp\left(4\nu_0 V \int_\Delta^\infty \frac{E\,dE}{\sqrt{E^2-\Delta^2}} \ln \frac{1-e^{-\frac{E}{T}}}{1+e^{-\frac{E}{T}}}\right) \approx$$

$$\approx \left(\tanh \frac{\Delta}{2T}\right)^{N_{eff}}$$

where $N_{eff} \approx 2\sqrt{2\pi\Delta T}\,\nu_0 V$ is the characteristic where $N_{eff} \approx 2\sqrt{2\pi\Delta T}\,\nu_0 V$ is the characteristic number of quasiparticles in the energy interval $\sim T$ above the gap.

$$\boxed{\delta\Omega \approx \Delta - T \underbrace{\ln N_{eff}}_{\text{Effective entropy}}}$$



lots of states here, which may make emerging a quasiparticle thermodynamically favourable

$$\boxed{T^* \sim \frac{\Delta}{\ln N_{eff}}}$$

– parity effect temperature

For a grain with $N \sim 10^9$ electrons $T^* \sim 0.3\,K$